Growth estimates of polynomials

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Abstract: Let us assume that $P : \mathbb{C}^n \to \mathbb{C}$ is a polynomial of degree m and that $|P| \leq 1$ on a subset X of $\mathbb{C}^n$. Which growth bounds of $|P|$ do we then have outside of X? If X is the unit ball in $\mathbb{C}^n$ in some complex norm $\|\cdot\|$, then it is easy to see that

$$|P(z)| \leq \max\{\|z\|^m, 1\}, \qquad z \in \mathbb{C}^n.$$

On a logarithmic scale this inequality is

$$\log |P(z)| \leq m \max\{\log \|z\|, 0\} = m \log^+ \|z\|, \qquad z \in \mathbb{C}^n.$$

For every subset X we define a function er V_X, which is called the *Green function of the set X with logarithmic pole at infinity* or the *Siciak-Zahariuta function of the set X*, such that

$$\log |P(z)| \leq mV_X(z), \qquad z \in \mathbb{C}^n,$$

for every polynomial P degree m with $|P| \leq 1$ on X. We have $V_X(z) = \log^+ \|z\| = \max\{\log \|z\|, 0\}$, $z \in \mathbb{C}^n$ if X is the unit ball in the norm $\|\cdot\|$. There are very few explicit examples for V_X other than those mentioned here.

In the lecture I will discuss a few new results on the properties of the functions V_X which I have achieved in collaboration with Finnur Lárusson University of Western Ontario in Canada (now at University of Adelaide in Australia) and has been published in our paper *The Siciak-Zahariuta extremal function as the envelope of disc functionals*, Annales Polonici Mathematici 86.2, 177-192, (2005). In the paper we show that if X is a domain in $\mathbb{C}^n$, then it is possible to express the function V_X with a so called disc formula, which is based on looking at all holomorphic maps $f = (f_1, \ldots, f_n)$ from the closed unit disc $\overline{\mathbb{D}}$ into $\mathbb{C}^n$ that map the unit circle $\mathbb{T}$ into X with poles in a finite subset, and investigating the location of the poles.